Resonance of a Rectangular Microstrip Patch on a Uniaxial Substrate

Kin-Lu Wong, *Member, IEEE,* Jeen-Sheen Row, Chih-Wen Kuo, *Member, IEEE,* and Kuang-Chih Huang

Abstract—**Effects of uniaxial anisotropy in the substrate on the complex resonant frequency of the microstrip patch antenna are investigated in terms of an integral equation formulation. The complex resonant frequency of the microstrip patch antenna is calculated by using Galerkin's method in solving the integral equation. The sinusoidal functions are selected as the basis functions, which show fast numerical convergence. Numerical results also indicate that both the resonant frequency and the half-power bandwidth are increased due to the positive uniaxial anisotropy and, on the other hand, decreased due to the negative uniaxial anisotropy.**

I. Introduction

The RESONANT frequency of the microstrip patch antenna, which has been found to be strongly dependent on the substrate permittivity, is a very important factor to be determined in microstrip antenna designs. It was also pointed out that many substrate materials used for printed-circuit antennas exhibit dielectric anisotropy, especially uniaxial anisotropy [1]–[2]. However, most related studies for the resonant frequencies of microstrip patch antennas concentrate on the isotropic substrate case [3]–[7]. Recently, the resonant frequency for the case of uniaxially anisotropic substrate is studied [8], which shows the results for resonant frequencies only. The information of the quality factor or the half-power bandwidth is not reported. In this paper we extend the study and use the approach of the integral-equation formulation for rigorous analysis of the complex resonant frequency of a rectangular microstrip patch on a uniaxial substrate. The Galerkin's method is employed to solve the integral equation for obtaining the complex resonant frequency, which provides the information of the resonant frequency and the half-power bandwidth of the patch antenna. Details of the theoretical treatment is presented in Sec. II. Various numerical results are shown in Sec. III. The selection of the basis function is first discussed. Both the effects of positive and negative uniaxial anisotropy on the complex resonant frequency of the patch antenna are shown. The half-power bandwidth is also calculated and analyzed. Conclusions are summarized in Sec. IV.

Manuscript received January 15, 1992; revised July 29, 1992.
This work was supported by the National Science Council, Republic of China, under Grant NSC81-0404-E110-004.
The authors are with the Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan 80424, Republic of China.
IEEE Log Number 9206297.

II. Theoretical Formulation of the Problem

Fig. 1 shows the geometry of a rectangular microstrip patch on a uniaxial substrate characterized by the free-space permeability μ_0 and the permittivity tensor $\overline{\overline{\epsilon}}$, where

$$\overline{\overline{\epsilon}} = \epsilon_0 \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_x & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{1}$$

The substrate material is uniaxially anisotropic with the optical axis along with the $\hat{z}$ axis; ϵ_0 is the free-space permittivity. A rectangular patch with length a and width b is printed on the grounded substrate, which has a uniform thickness of d. The surface current $\vec{J}$ on the patch can be expanded into a series of basis functions $\vec{J}_{xn}$ and $\vec{J}_{ym}$, i.e.

$$\vec{J} = \sum_{n=1}^{N} I_{xn}\vec{J}_{xn} + \sum_{m=1}^{M} I_{ym}\vec{J}_{ym}, \tag{2}$$

where I_{xn} is the unknown coefficient to be determined for the nth expansion mode of the surface current in the x direction and I_{ym} is for the mth expansion mode in the y direction. A Fourier transform of $\vec{J}_{xn}$ and $\vec{J}_{ym}$ can be written as

$$\vec{F}_{xn}(k_x, k_y) = \int_x \int_y \vec{J}_{xn}(x, y) e^{-j(k_x x + k_y y)}\, dx\, dy, \tag{3a}$$

$$\vec{F}_{ym}(k_x, k_y) = \int_x \int_y \vec{J}_{ym}(x, y) e^{-j(k_x x + k_y y)}\, dx\, dy, \tag{3b}$$

where k_x and k_y are the wavenumbers in the x and y directions, respectively. The integral equation describing the field $\vec{E}$ on the patch can be expressed to be

$$\vec{E} = \frac{-j30}{\pi k_0} \int_x \int_y \overline{\overline{Q}} \cdot \vec{F} e^{-j(k_x x + k_y y)}\, dk_x\, dk_y, \tag{4}$$

where $\overline{\overline{Q}} = \hat{x}Q_{xx}\hat{x} + \hat{x}Q_{xy}\hat{y} + \hat{y}Q_{yx}\hat{x} + \hat{y}Q_{yy}\hat{y}$ and $\vec{F} = \sum_{n=1}^{N} I_{xn}\vec{F}_{xn} + \sum_{m=1}^{M} I_{ym}\vec{F}_{ym}$; $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, ω is the wave angular frequency. The quantity Q represents the Green's function and has been derived and given in [2]. The first subscript for Q shows the x or y components of the generated electric field and the second subscript is the orientation of the infinitesimal electric dipole on the surface of the uniaxial substrate. By applying the Galerkin's method [9], the electric field integral equation of (4) can be discretized into the following matrix equation:

$$\begin{bmatrix} (Z^{xx}_{kn})_{N\times N} & (Z^{xy}_{km})_{N\times M} \\ (Z^{yx}_{ln})_{M\times N} & (Z^{yy}_{lm})_{M\times M} \end{bmatrix} \cdot \begin{bmatrix} (I_{xn})_{N\times 1} \\ (I_{ym})_{M\times 1} \end{bmatrix} = \begin{bmatrix} (V_{xn})_{N\times 1} \\ (V_{ym})_{M\times 1} \end{bmatrix}, \tag{5}$$

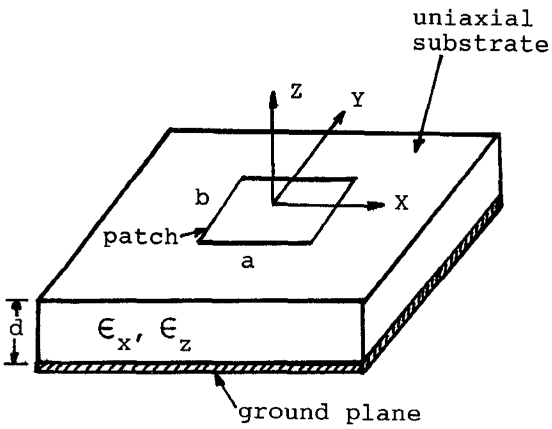


Fig. 1. Geometry for a rectangular patch on a uniaxial substrate

where

$$Z_{kn}^{xx} = \frac{-j30}{\pi k_0} \iint_{-\infty}^{\infty} F_{xk}^{*} Q_{xx} F_{xn}\, dx_x\, dk_y, \tag{6a}$$

$$Z_{km}^{xy} = \frac{-j30}{\pi k_0} \iint_{-\infty}^{\infty} F_{xk}^{*} Q_{xy} F_{ym}\, dk_x\, dk_y, \tag{6b}$$

$$Z_{ln}^{yx} = \frac{-j30}{\pi k_0} \iint_{-\infty}^{\infty} F_{yl}^{*} Q_{yx} F_{xn}\, dk_x\, dk_y, \tag{6c}$$

$$Z_{lm}^{yy} = \frac{-j30}{\pi k_0} \iint_{-\infty}^{\infty} F_{yl}^{*} Q_{yy} F_{ym}\, dk_x\, dk_y, \tag{6d}$$

$$F_{xk}^{*} = F_{xk}(-k_x, -k_y),$$
$$F_{yl}^{*} = F_{yl}(-k_x, -k_y),$$

and

$$k, n = 1, 2, \cdots, N,$$
$$l, m = 1, 2, \cdots, M.$$

Since the resonant frequencies are defined to be the frequencies at which the field and the current can sustain themselves without a driving source, i.e. V_{xn} and V_{ym} in (5) are vanished. Therefore, for the existence of nontrivial solutions, the determinant of the $[Z]$ matrix must be zero. This condition is satisfied by a complex frequency $f = f' + jf$ that gives the resonant frequency f' and the half-power bandwidth $2f/f'$ of the patch antenna.

III. Numerical Results and Discussion

The basis functions $\vec{J}_{xn}$ and $\vec{J}_{ym}$ for the following numerical calculations are selected to be sinusoidal functions of

$$\vec{J}_{xn} = \hat{x} \sin\left[\frac{p\pi}{a}\left(x + \frac{a}{2}\right)\right] \cdot \cos\left[\frac{q\pi}{b}\left(y + \frac{b}{2}\right)\right], \tag{7a}$$

$$\vec{J}_{ym} = \hat{y} \sin\left[\frac{r\pi}{b}\left(y + \frac{b}{2}\right)\right] \cdot \cos\left[\frac{s\pi}{a}\left(x + \frac{a}{2}\right)\right], \tag{7b}$$

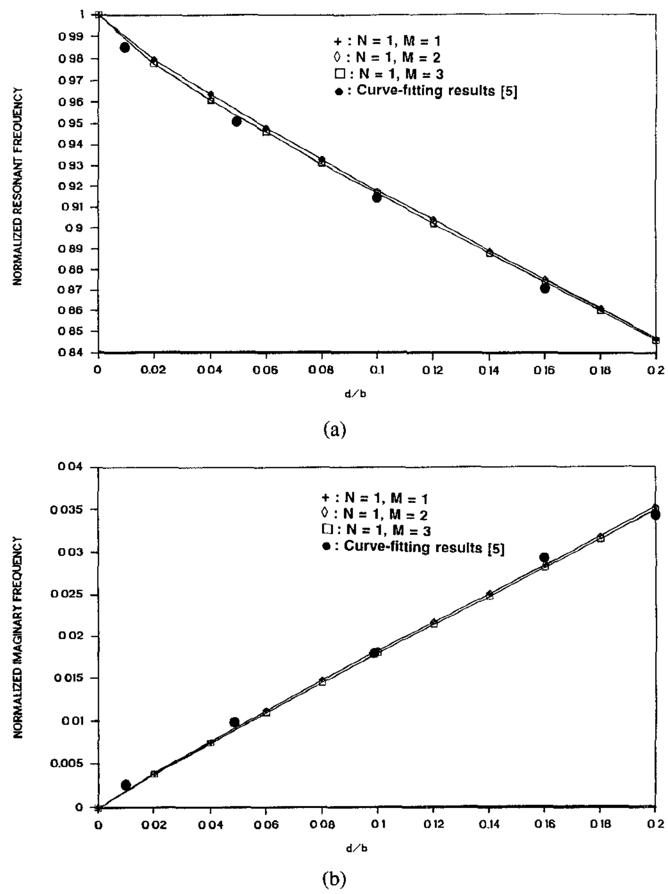


Fig. 2. Frequency shifts for different numbers of basis functions; $\epsilon_x = \epsilon_z = 2.35, a = 1.5$ cm, $b = 1.0$ cm. (a) Real frequency, (b) Imaginary frequency.

where p, q, r and s are integers. The combination of p, q, r and s depends on the mode numbers n and m. For the first three modes, $n = 1, 2$, and 3, the values of (p, q) are (1, 0), (1, 1), and (2, 1), respectively, and the values of (r, s) are (1, 0), (1, 1), and (1, 2) for $m = 1, 2$, and 3. The basis currents J_{xn} and J_{ym} then vanish at $x = \pm a/2$ and $y = \pm b/2$, respectively.

Fig. 2 shows the calculation results for different numbers of basis functions with $\epsilon_x = \epsilon_z = 2.35, a = 1.5$ cm, and $b = 1.0$ cm; N is selected to be 1 and M is varied from 1 to 3. The substrate thickness is normalized by the patch width b and the frequency is normalized to be unity when the substrate thickness approaching zero. The mode studied here is the TM_{010} mode, the dominant mode for the current in the y direction. The time dependence of the wave is assumed to be $e^{j\omega t}$. For the results shown in Fig. 2, the numerical calculation can be seen to almost converge for $N = 1, M = 1$. The results for $N = 1, M = 2$ (shown by the line with open rhombus) are almost the same as those for $N = 1, M = 1$ (the cross signs), and the results for $N = 1, M = 3$ (the line with open rectangles) are only slightly different from those for $N = 1, M = 1$. Hence, for the rest of the studies, we will use the basis functions of (7) with $N = 1, M = 1$ for numerical calculations. It is estimated that the computation time for $N = 1, M = 1$ on a HP720 workstation is about 360 seconds, which is only about 50% of that for the case of $N = 1, M = 2$. Furthermore, it is found that the most

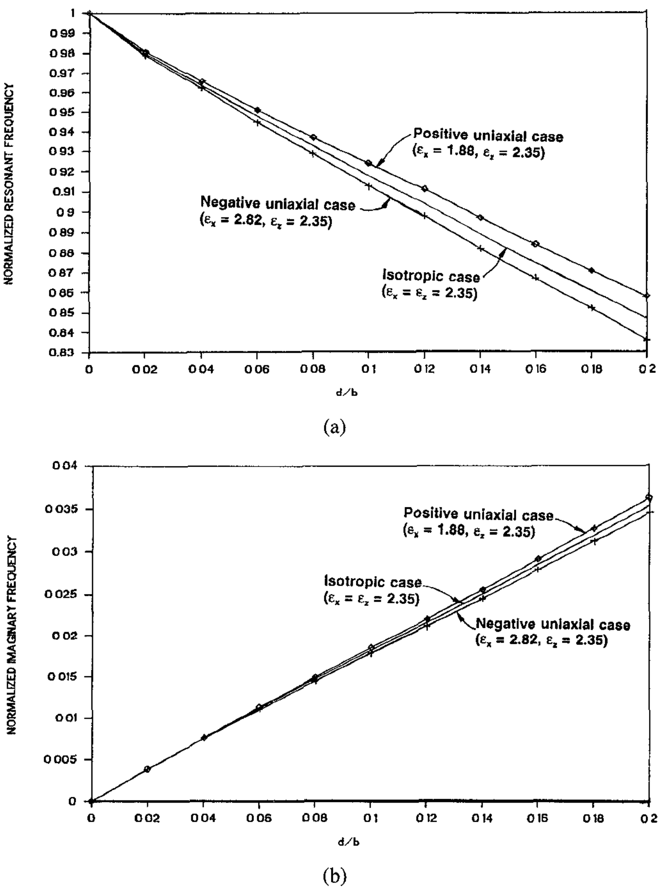


Fig. 3. Real and imaginary frequency shifts versus the substrate thickness for the isotropic ($\epsilon_x = \epsilon_z = 2.35$), positive uniaxial ($\epsilon_x = 1.88, \epsilon_z = 2.35$), and negative uniaxial ($\epsilon_x = 2.82, \epsilon_z = 2.35$) substrates; $a = 1.5$ cm, $b = 1.0$ cm. (a) Real frequency, (b) Imaginary frequency.

time-consuming computation for the resonant frequency is the calculation of the impedance matrix element $Z_{kn}^{xx}, Z_{km}^{xy}, Z_{ln}^{yx}$ and Z_{lm}^{yy} in (5), which involves double integral computations. From the numerical results it is found that the values of Z_{km}^{xy} and Z_{ln}^{yx} are on the order of about 10^{-7}–10^{-9} times those of Z_{kn}^{xx} and Z_{lm}^{yy}. Therefore, the computation of Z_{km}^{xy} and Z_{ln}^{yx} can be omitted which results in almost no difference in the obtained results for the resonant frequency. The computation time for the case of $N = 1, M = 1$ with omitting the calculation of Z_{km}^{xy} and Z_{ln}^{yx} is further reduced to be only about 240 seconds on the HP720 workstation for the calculation of one resonant frequency. Finally, the convergent solutions of our calculation are also compared with the numerical results obtained from the curve-fitting formula in [5], which are shown by solid circles in Fig. 2. It can be seen that our numerical calculations agree well with the results obtained from [5]. This validates our calculations here and the following results for the complex resonant frequencies due to the uniaxial anisotropy in the substrate.

In Fig. 3 the real and imaginary frequency shifts versus the substrate thickness (normalized by the patch width b) are studied. The solid line represents the results for the isotropic case ($\epsilon_x = \epsilon_z = 2.35$), while the line with open rhombus is for the positive uniaxial substrate (Anisotropic ratio AR = ϵ_x/ϵ_z = 1.88/2.35 = 0.8) and the line with cross signs is for the negative uniaxial substrate (AR = 2.82/2.35 = 1.2). It is found that the real frequency, i.e. the resonant frequency, is shifted to higher frequencies for the positive uniaxial case and, on the other hand, is shifted to lower frequencies for the negative uniaxial case. As for the imaginary frequency, it is increased due to the positive uniaxial anisotropy and decreased due to the negative uniaxial anisotropy. It can also be seen that the higher the substrate thickness, the higher the frequency shifts due to the uniaxial anisotropy. Fig. 4 shows the results for the

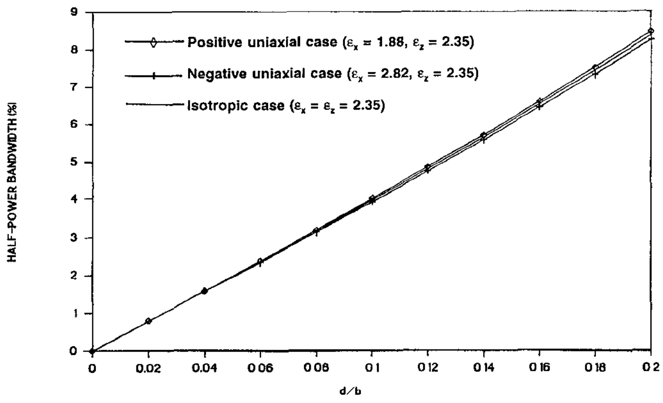


Fig. 4. Variations of the half-power bandwidth with the substrate thickness for the case in Fig. 3.

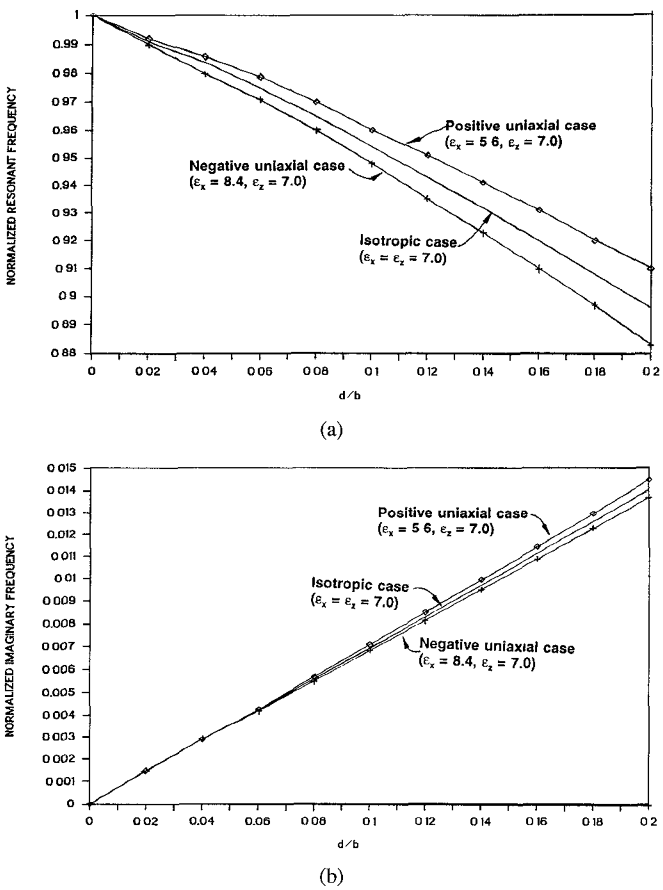


Fig. 5. Real and imaginary frequency shifts versus the substrate thickness for the isotropic ($\epsilon_x = \epsilon_z = 7.0$), positive uniaxial ($\epsilon_x = 5.6, \epsilon_z = 7.0$), and negative uniaxial ($\epsilon_x = 8.4, \epsilon_z = 7.0$) substrates; $a = 1.5$ cm and $b = 1.0$ cm. (a) Real frequency, (b) Imaginary frequency.

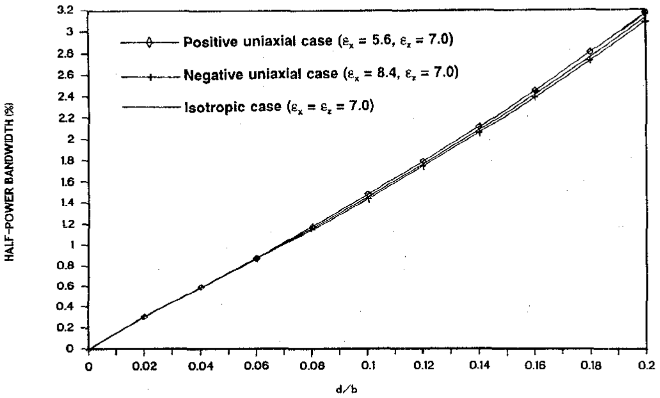


Fig. 6. Variations of the half-power bandwidth with the substrate thickness for the case in Fig. 5.

half-power bandwidth of the patch antenna, calculated from $2f/f'$. The positive uniaxial anisotropy slightly increases the half-power bandwidth, while the negative uniaxial anisotropy slightly decreases the half-power bandwidth. The variations of the bandwidth due to the uniaxial anisotropy are also seen to increase when the substrate thickness is increased. In Fig. 5 the case of higher permittivity with $\epsilon_z = 7.0$ is also studied. Fig. 6 shows the results of the half-power bandwidth for the case in Fig. 5. The frequency shifts and the variations of the bandwidth due to the uniaxial anisotropy can be seen to be the same as discussed in Figs. 3 and 4 with $\epsilon_z = 2.35$.

IV. Conclusions

The resonant frequency and half-power bandwidth of a rectangular patch antenna are studied using the integral equation formulation. The sinusoidal basis functions for calculating the current on the patch show fast numerical convergence. The obtained results show that the uniaxial anisotropy effect on the complex resonant frequencies increases with the increasing of the substrate thickness. In general, the resonant frequencies shift to higher or lower frequencies for positive or negative uniaxial anisotropy in the substrate material. The positive uniaxial anisotropy is also found to slightly increase the half-power bandwidth, while the negative uniaxial anisotropy slightly decreases the half-power bandwidth.

References

[1] N. G. Alexopoulos, "Integrated-circuit structures on anisotropic substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 847–881, Oct. 1985.
[2] D. M. Pozar, "Radiation and scattering from a microstrip patch on a uniaxial substrate," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 613–621, June 1987.
[3] Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and experiment on microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 137–145, Mar. 1979.
[4] T. Itoh and W. Menzel, "A full-wave analysis method for open microstrip structures," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 63–67, Jan. 1981.
[5] W. C. Chew and Q. Liu, "Resonance frequency of a rectangular microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1045–1056, Aug. 1988.
[6] Q. Liu and W. C. Chew, "Curve-fitting formulas for fast determination of accurate resonant frequency of circular microstrip patches," *Proc. Inst. Elec. Eng.*, pt. H, vol. 135, pp. 289–292, Oct. 1988.
[7] K. M. Luk, W. Y. Tam, and C. L. Yip, "Analysis of circular microstrip antennas with superstrate," *Proc. Inst. Elec. Eng.*, pt. H, vol. 136, pp. 261–262, June 1989.
[8] R. M. Nelson, D. A. Rogers, and A. G. D'Assuncao, "Resonant frequency of a rectangular microstrip patch on several uniaxial substrates," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 973–981, July 1990.
[9] R. F. Harrington, *Field Computation by Moment Method.* New York: Macmillan, 1968.

Kin-Lu Wong (M'91) was born in Tainan, Taiwan, in 1959. He received the B.S. degree in electrical engineering from National Taiwan University, Taipei, Taiwan, in 1981, and the M.S. and Ph.D. degrees in electrical engineering from Texas Tech University, Lubbock, TX, in 1984 and 1986, respectively.

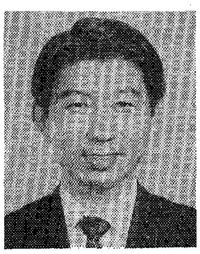
From 1986 to 1987 he was a visiting Scientist with Max-Planck-Institut fuer Plasmaphysik in Munich, Germany. In 1987 he was with the Department of Electrical Engineering at National Sun Yat-Sen University, Kaoshiung, Taiwan, where he became a Professor in 1991. His research interests include electromagnetic theory, antenna design, and plasma science.

Jeen-Sheen Row was born in Miaoli, Taiwan, Republic of China, in 1967. He received the B.S. degree in electrical engineering from Tamkang University, Taipei, Taiwan, in 1989.

Since 1990 he has been working toward the Ph.D. degree in the Department of Electrical Engineering at the National Sun Yat-Sen University, Kaohsiung. His current research interests are in antenna theory and design, and EM wave propagation.

Chih-Wen Kuo (S'88–M'91) was born in Kaohsiung, Taiwan, R.O.C., on June 11, 1964. He received the B.S. degree in electrical engineering from National Taiwan University, Taipei, Taiwan, in 1986 and the M.S. and Ph.D. degrees in electrical engineering from the University of Texas at Austin in 1989 and 1991, respectively.

In August 1991, he joined the faculty at National Sun Yat-Sen University, Kaohsiung, Taiwan, as an associate professor of the Electrical Engineering Department. His research interests involve the loss calculations of planar transmission line structures and the characterization of discontinuities between planar transmission lines.



Kuang-Chih Huang was born in Chiayi, Taiwan, Republic of China, in 1939. He received the B.S. Degree in electrical engineering from National Cheng Kung University, Tainan, Taiwan, in 1962, the M.S. degree in electronic engineering from National Chiao Tung University, Hsinchu, Taiwan, in 1967, and the Ph.D. degree in electrophysics from Polytechnic Institute of Brooklyn, NY, in 1970.

From 1970 to 1971 he was a Post Doctor Research Fellow with the Polytechnic Institute of Brooklyn. In 1971 he joined the faculty of electrophysics at the National Chiao Tung University. Dr. Huang was appointed as the Dean of Academic Affairs of National Chiao Tung University in 1978. In 1986, he became the Dean of Engineering at National Sun Yat-Sen University. Early in 1991, he was appointed by the Minister of Education of ROC as the President of the National Kaohsiung Institute of Technology. Besides the academic administration of high education, his research interests include EM wave propagation, laser theory, plasma science, and semi-conductor physics and devices.